Applications of Quadratic Functions:

1. If the flight of an object is defined by the equation $y = -16x^2 + 64x$ where y represents distance and x represents time, determine the following:
    a) the time at which the object reaches it maximum height

    $$x = -\frac{b}{2a} \Rightarrow x = -\frac{64}{2(-16)} = 2$$

    b) the maximum height the object reaches

    $$y = \frac{4ac - b^2}{4a} \Rightarrow y = \frac{4(-16)(0) - (64)^2}{4(-16)} = \frac{0 - 4096}{-64} = 64$$

    c) the time it takes for the object to reach the ground

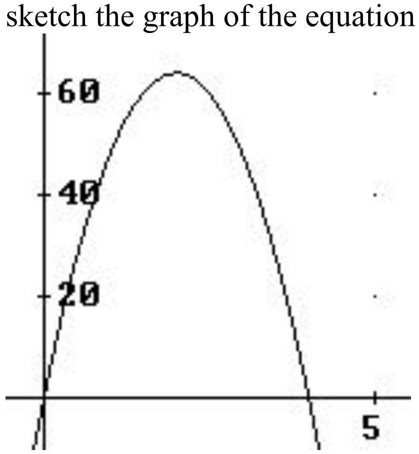
    since a parabolic curve and if it rearches max height in 2 sec then it takes 2 sec to return to the ground $\Rightarrow$ therefore a total of 4 sec

    or

    $0 = -16x^2 + 64x \Rightarrow$ because y = 0 when object hits the ground

    $0 = -16x(x-4) \Rightarrow$ therefore $x = 0$ or $x = 4$

    d) sketch the graph of the equation (use appropriate scale)



2. Use the following equation to solve the problems posed below:

    If an object is thrown vertically upwards with a starting speed of v meters per second, from an altitude of h meters, then the height y after x seconds is given by $y = -4.9x^2 - vx + h$

    a) A ball is thrown upward with a velocity of 14.7 m/s by a person 1.4 m tall.

    $y = -4.9x^2 - (-14.7)x + 1.4$ because the velocity decreases as the object moves to a maximum height $\Rightarrow y = -4.9x^2 + 14.7x + 1.4$

    1) What is the maximum height reached by the ball?

    $$y = \frac{4ac - b^2}{4a} \Rightarrow y = \frac{4(-4.9)(1.4) - (14.7)^2}{4(-4.9)} = \frac{-27.44 - 216.09}{-19.6} = 12.425$$

    2) How long does it take for the ball to reach a maximum height?

$$x = -\frac{b}{2a} \Rightarrow x = -\frac{14.7}{2(-4.9)} = 1.5$$

3) How long is the ball in the air before it strikes the ground?

because it returns to the ground y = 0 or we determine an x-intercept

we have two procedures a) factoring or b) quadratic formula

$$y = -4.9x^2 + 14.7x + 1.4 \Rightarrow 0 = -4.9x^2 - 14.7x + 1.4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(14.7) \pm \sqrt{(14.7)^2 - 4(-4.9)(1.4)}}{2(-4.9)} =$$

$$\frac{-14.7 \pm \sqrt{216.09 + 27.44}}{-9.8} = \frac{-14.7 \pm \sqrt{243.53}}{-9.8} = \frac{-14.7 \pm 15.6}{-9.8} =$$

$$\frac{-14.7 + 15.6}{-9.8} = -0.09 \text{ or } \frac{-14.7 - 15.6}{-9.8} = 3.09$$

b) A missile is fired vertically with a velocity of 2450 m/s from a base 500 m above sea level.

$y = -4.9x^2 - (-2450)x + 500$ because the velocity decreases as the object moves to a maximum height $\Rightarrow y = -4.9x^2 + 2450x + 500$

1) What is the maximum height reached by the missile?

$$y = \frac{4ac - b^2}{4a} \Rightarrow y = \frac{4(-4.9)(500) - (2450)^2}{4(-4.9)} = \frac{-9800 - 6002500}{-19.6} = 306750$$

2) How long does it take to reach the maximum height?

$$x = -\frac{b}{2a} \Rightarrow x = -\frac{2450}{2(-4.9)} = 250$$

3) How long will it be before the missile descends to an altitude of 500 m above sea level?

because it returns to the ground y = 0 or we determine an x-intercept

we have two procedures a) factoring or b) quadratic formula

$$y = -4.9x^2 + 2450x + 500 \Rightarrow 0 = -4.9x^2 + 2450x + 500$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2450) \pm \sqrt{(2450)^2 - 4(-4.9)(500)}}{2(-4.9)} =$$

$$\frac{-2450 \pm \sqrt{6002500 + 98000}}{-9.8} = \frac{-2450 \pm \sqrt{6100500}}{-9.8} = \frac{-2450 \pm 2469.9}{-9.8} =$$

$$\frac{-2450 + 2469.9}{-9.8} = -2.03 \text{ or } \frac{-2450 - 2469.9}{-9.8} = 502.03$$

c) A diver jumps from a tower 30 m above the water with a velocity of 4.9 m/s. How long does it take for the diver to reach a point 0.6m above the water?

$y = -4.9x^2 - vx + h \Rightarrow y = -4.9x^2 - 4.9x + 30$

since we want the time it takes to reach a point 0.6m above the water we set y = 0.6

$0.6 = -4.9x^2 - 4.9x + 30 \Rightarrow 0 = -4.9x^2 - 4.9x + 29.4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4.9) \pm \sqrt{(-4.9)^2 - 4(-4.9)(30)}}{2(-4.9)} =$$

$$\frac{4.9 \pm \sqrt{24.01 + 588}}{-9.8} = \frac{4.9 \pm \sqrt{612.01}}{-9.8} = \frac{4.9 \pm 24.73}{-9.8} =$$

$$\frac{4.9 + 24.73}{-9.8} = -3.02 \text{ or } \frac{4.9 - 24.73}{-9.8} = 2.02$$

3. A pilot was crop dusting in his single engine plane at an altitude of 50 m when the propeller fell off. The height, h, of a falling object is given by $h = A - 4.9t^2$ where A is the initial height of the object and t is the time elapsed.
   a) How far above the ground is the propeller after 3s?

      $h = A - 4.9t^2 \Rightarrow h = 50 - 4.9t^2$

      $h = 50 - 4.9(3)^2 \Rightarrow h = 50 - 44.1 = 5.9$

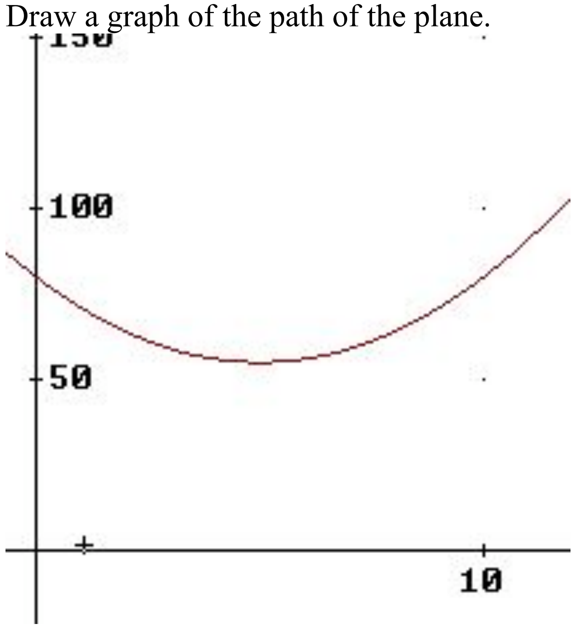
   b) Will the propeller have hit the ground after the fourth second?

      $h = A - 4.9t^2 \Rightarrow h = 50 - 4.9t^2$

      $0 = 50 - 4.9(t)^2 \Rightarrow 10.2 = t^2 \Rightarrow t = \pm 3.19$

      $\therefore$ the propeller will have hit the ground after 4 sec, in fact it makes contact at 3.19 seconds.

4. During a stunt, the power dive of a plane is given by the equation, $h = t^2 - 10t + 80$ where h (in meters) is the height of the plane after time t (in seconds)
   a) Draw a graph of the path of the plane.



b) How high is the plane at the start of the dive?

$h = t^2 - 10t + 80 \;\; \Rightarrow \text{time} = 0$

$h = 80$

c) How high above ground level is the plane at its minimum point?

$h = t^2 - 10t + 80$

$$y = \frac{4ac - b^2}{4a} \Rightarrow y = \frac{4(1)(80) - (-10)^2}{4(1)} = \frac{320 - 100}{4} = \frac{220}{4} = 55$$